

5.2

Special Products



The patterns in tile floors, brick patios, and quilts often repeat. Some quilts are made from square pattern blocks of material that are stitched together. The square pattern block shown starts with a square that is enlarged with the addition of rectangles to each of its dimensions until the desired square dimensions are reached. If the side length of the square pattern block is represented by $x + 3$, what is an expression for its area?



Tools

Optional

■ algebra tiles

Investigate

How can you use patterns to find special products?

A: Squaring Binomials

Method 1: Use Pencil and Paper

1. Use algebra tiles or a diagram to square the binomial.

$$(x + 3)^2 = (x + 3)(x + 3)$$

2. Expand and simplify.

a) $(x + 3)^2$

b) $(x + 2)^2$

c) $(x - 6)^2$

d) $(x - 4)^2$

e) $(2x + 5)^2$

f) $(3x - 1)^2$

g) $(2x - 5y)^2$

h) $(4x + 7y)^2$

3. Consider each simplified expansion from step 2.

- a) How is the first term in each trinomial related to the first term in each binomial?
- b) How is the last term in each trinomial related to the last term in each binomial?
- c) How is the middle term in each trinomial related to the terms in the binomial?

4. **Reflect** Write a rule for expanding and simplifying $(a + b)^2$ or $(a - b)^2$.

5. Use your rule to square each binomial.

a) $(5x + 3y)^2$

b) $(7c - 4d)^2$

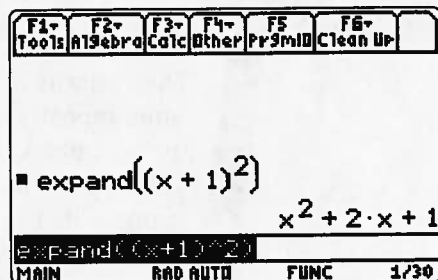
Method 2: Use a Computer Algebra System (CAS)

1. Clear the calculator's memory by selecting **2:NewProb** from the **Clean Up** menu.

Tools
■ TI-89 calculator

2. Use the **Expand** function on each square of a binomial. Record the results.

- a) $(x + 1)^2$ b) $(x + 2)^2$
c) $(x + 3)^2$ d) $(x + 4)^2$
e) $(x + 5)^2$



3. Compare the individual terms of the expansion to the binomial. Describe any patterns you notice, including any for the signs of the terms in the trinomial.
4. Use a CAS to expand each square of a binomial. Record the results.

a) $(2x + 2)^2$ b) $(2x + 3)^2$
c) $(2x - 4)^2$ d) $(2x - 5)^2$
5. Compare the individual terms of the expansion to the binomial. Describe any patterns you notice. How has your description of patterns changed compared to step 3?
6. Use a CAS to expand each square of a binomial. Record the results.

a) $(3a + 2)^2$ b) $(5m - 3)^2$
c) $(4 + 2b)^2$ d) $(7 - 3z)^2$
e) $(2x + 3y)^2$
7. Compare the individual terms of the expansion to the binomial. Have the patterns you described in step 5 changed?
8. **Reflect** Write a rule for expanding each square of a binomial.

a) $(a + b)^2$
b) $(a - b)^2$

B: Product of a Sum and a Difference of Two Terms

Method 1: Use Pencil and Paper

1. Expand and simplify.

a) $(x + 3)(x - 3)$
b) $(2y + 5)(2y - 5)$
c) $(x - 4)(x + 4)$
d) $(3k - 7)(3k + 7)$

Example 1 Apply Special Product Patterns

Expand and simplify.

- a) $(x + 4)^2$ b) $(k - 5)^2$ c) $(3y + 7x)^2$
 d) $(q - 11)(q + 11)$ e) $(4m + 3n)(4m - 3n)$

Solution

a) $(a + b)^2 = a^2 + 2ab + b^2$ Use the appropriate pattern for squaring a binomial.
 $(x + 4)^2 = (x)^2 + 2(x)(4) + (4)^2$
 $= x^2 + 8x + 16$

b) $(a - b)^2 = a^2 - 2ab + b^2$ Use the appropriate pattern for squaring a binomial.
 $(k - 5)^2 = (k)^2 - 2(k)(5) + (5)^2$
 $= k^2 - 10k + 25$

c) $(a + b)^2 = a^2 + 2ab + b^2$ Use the appropriate pattern for squaring a binomial.
 $(3y + 7x)^2 = (3y)^2 + 2(3y)(7x) + (7x)^2$
 $= 9y^2 + 42xy + 49x^2$

d) $(a - b)(a + b) = a^2 - b^2$ Use the pattern for the product of a sum and a difference.
 $(q - 11)(q + 11) = (q)^2 - (11)^2$
 $= q^2 - 121$

e) $(a + b)(a - b) = a^2 - b^2$ Use the pattern for the product of a sum and a difference.
 $(4m + 3n)(4m - 3n) = (4m)^2 - (3n)^2$
 $= 16m^2 - 9n^2$



Example 2 Helicopter Pad

The radius of a circular helicopter landing pad is increased by 3 m.

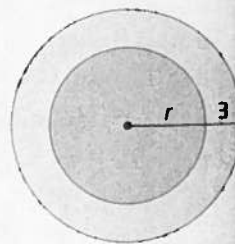
- a) Find a simplified expression for the area of the new circle.
 b) Find a simplified expression for the increase in area.

Solution

a) Area of the original circle = πr^2

Area of the larger circle = $\pi(r + 3)^2$
 $= \pi[(r)^2 + 2(r)(3) + (3)^2]$
 $= \pi(r^2 + 6r + 9)$
 $= \pi r^2 + 6\pi r + 9\pi$

I can use the appropriate pattern for squaring a binomial.



b) Increase in area = (area of larger circle) - (area of original circle)
 $= (\pi r^2 + 6\pi r + 9\pi) - (\pi r^2)$
 $= 6\pi r + 9\pi$

